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A Gravity Forward Modeling Method based on Multiquadric Radial Basis Function

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It is one of the most important part to build an accurate gravity model in geophysical exploration. Traditional gravity modelling is usually based on grid method, such as difference method and finite element method widely used. Due to self-adaptability lack of division meshes and the difficulty of high-dimensional calculation, the accuracy of those grid method with uniform size orthogonal mesh will be limited when the boundary shape of solution domain is more complex, especially in continuous mesh reconstruction or large deformation, and even can not be solved (Nguyen et al., 2008). Meshfree method is a nodal numerical method and showing huge advantages in many fields, which avoids cumbersome mesh generation and provides a new method for numerical problem solving (Liu et al., 2005).

Radial basis function method is a kind of typical meshfree method, which has infinite smooth add function on a coordinate direction derivative, and can be said for all the discrete points along the direction function values of the weighted linear combination (Buhmann, 2000; Wendland, 2005). It can solve all kinds of partial differential equations (such as elliptic, parabolic and hyperbolic) problem with good results. After more than 20 years of development, the radial basis method (RBF) has been applied to many fields successfully. However, the application of RBF and even meshless method in gravity and magnetic exploration and other geophysical methods is quite rare. Internationally, Yoshikazu Tanaka (2011) applied radial point interpolation to transient electromagnetic fields, Fornberg (2015) carried out preliminary geological modeling of radial basis function-finite difference in spherical and finite domains, and Franciane et al. (2017) applied adaptive meshless parameterization technology to full waveform inversion. At present, only the elastic wave field (Jia et al., 2005, 2006a, 2010; Wang et al., 2007; Liu et al., 2020), Radar wave field (Feng et al., 2013), magnetotelluric field (Su et al., 2012; Yan et al., 2014; Ji et al., 2016; Lu et al., 2017; Yan et al., 2019) and the continuation and forward modeling of gravity and magnetic potential fields (Kong et al., 2017; Li et al., 2018) has achieved certain results. Therefore, this paper carry out the approximate calculation research of the gravity forward model based on MQ radial basis function, so as to make full use of prior information and improve efficiency.

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Traditional gravity forward modeling method

The most classical method of gravity forward modeling is to use the formula of Universal Gravitation:

$$\Delta g=\frac{\partial v}{\partial z}=G\iiint_v \frac{\sigma(\zeta-z)d\xi d\eta d\zeta}{[(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2]^{3/2}} \tag{1}$$

Divided the underground space of geologic body into two or three dimensions, and then calculated the gravity value of each unit by formula (1), the forward calculation of global gravity is completed by calculating the accumulative gravity values of all position points in the observation region one by one. Although the point-by-point calculation method can obtain the accurate gravity value of the working area, it is extremely time-consuming, which is not good for the time requirement of repeated forward and inverse in large-scale calculation, and the precision control is also complicated.

Point interpolation radial basis function to gravity forward modeling

The method of obtaining gravity values by point interpolation radial basis function is as follows: Select a polynomial function as the linear basis function, such as $P^T(x,y) = \{1,x,y,x^2,xy,y^2,x^3,x^2y,y^3\}$, calculate the gravity values of a small number of random positions, and then calculate the coefficient vector C according to the formula $C = P^{-1}U$, where U is a series of calculated gravity values corresponding to $P(X,Y)$ position points. Then calculate according to the following formula.

$$U(X,Y)=P^T(X,Y)P^{-1}U \tag{2}$$

where $U(X,Y)$ is the gravity value of the unknown point, and $P^T(X,Y)$ is the corresponding basis function matrix.

Compared with the traditional mesh method, the point interpolation method can get the required gravity distribution with less computation. In the process of forward inversion, the point interpolation method can not only save the operation time, but also can artificially select the position and number of discrete points to do multi-scale analysis and local analysis, so as to reduce the number of inversion parameters and the ill-posed degree of gravity inversion. But after testing, it was found to be

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less accurate. In particular, the calculation method of the basis function is a power function, so that the value in the distance from the origin of the grid will be very large, resulting in poor stability. However, the actual underground space will not be different because of the selected location of the origin of coordinates, and its value will only be related to the distance between the geological body and the observation point. In conclusion, the accuracy of pure point interpolation method is still limited.

MQ radial basis function to gravity forward modeling method

The workflow of the gravity model forward modeling method based on MQ radial basis function is designed as follows: Divide the underground space of the working area into blocks and arrange nodes; According to the problem to be solved, the parameters of radial basis function are adjusted.The actual data collected or the calculated model data were substituted into the functional equation with the radial basis function as the shape function, and the weight coefficient vector was solved. The actual data included the sampled data and the logging data. The weight coefficient vector and the points with known information are used to solve the information of global grid points, and finally the global gravity field distribution is obtained to complete the forward modeling. The workflow is shown in Fig. 1. The details are as follows:

(1) Discretization of subdomains according to nodes

The model of the study area is shown in Fig. 2, where AA′ is the surface observation surface. A series of known source Pi values are obtained from surface data and logging data, and the influence domain of these Pi values is spread to each grid node to calculate the entire solution domain. Chebyshev collocation method was optimized by experiment. Chebyshev nodes defined in the solution interval are taken as the collocation points in the calculation, and boundary points are added to form the collocation points of the closed interval (Fig. 2).

Selection of shape parameters c and θ of the radial basis function of MQ

The form of MQ radial basis function is as follows:

$$g_I(x) = (r_I^2 + c^2)^{\theta - \frac{3}{2}} \tag{3}$$

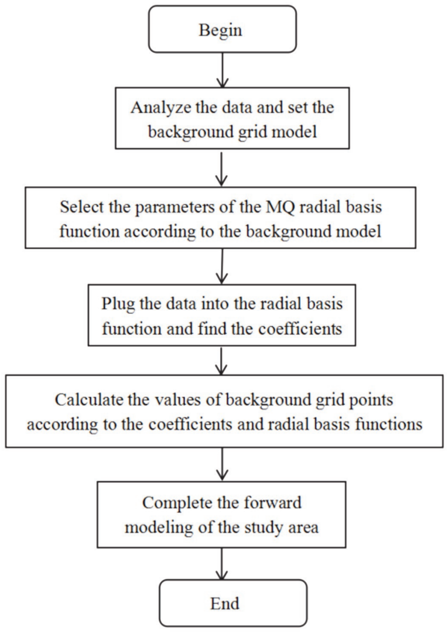

Fig. 1. Flow chart of gravity forward modeling.

where r_I is the norm of calculated points and nodes, and c and θ need to be selected flexibly according to the actual situation (Hardy, 2017).

In order to select appropriate shape parameters c and θ, we carried out model tests, and the calculated results shows that the influence of c on MQ function is reflected in the size of the influence domain. When c is small, MQ function is sharper, which makes the node's influence domain smaller, resulting in insufficient utilization of node information. When c is too large, although the influence domain of nodes can be enlarged, in practice, nodes often do not have a large influence scope. Therefore, in previous studies, nodes are often divided into supporting domains. Under the premise that the support domain has been divided, the choice of large c value will cause serious numerical dispersion and will not get ideal results. c is selected according to the empirical parameters and the distribution law of background grid, i.e., the product of

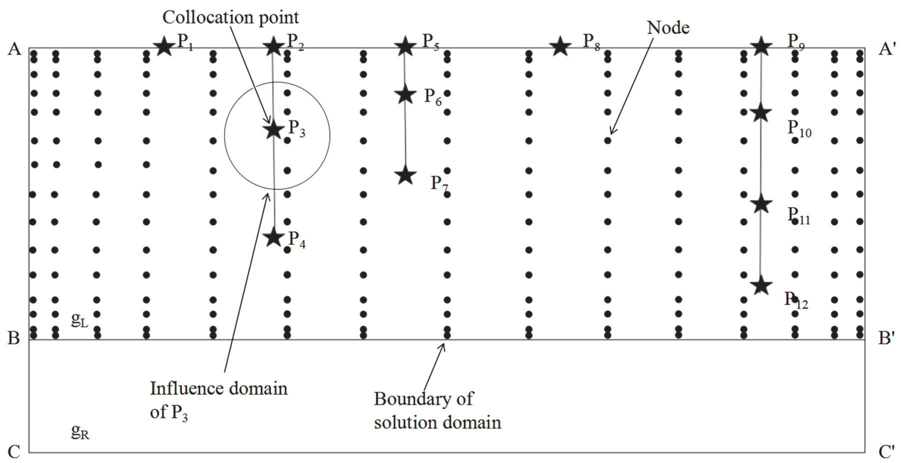

Fig. 2. Model schematic diagram with collocation point and node design of study area.
AA′ represents the surface, the depth is 0 meter; BB′ represents the base, with a depth of 5000 meters; CC′ is the core. g_L is the local field and g_R is the regional field; P_n (n = 1, 2,...N) is the source point of known information.

grid spacing 12.5 and empirical parameter 1.789. Similarly, different values of θ represent different types of radial basis functions of MQ. The selection of θ does not have a significant impact on the shape of the function. In addition, exponentials in the derived formulas of gravity theory have strict physical meaning, so it is not suitable to make great adjustments to the exponentials in the shape function, so $\theta = 1$ is taken.

(3) Block sparse matrix was constructed to solve the deformation equation

The equation of the radial basis function is:

$$u(X) \approx u^*(X) = \sum_{i=1}^{n} R_i(X) b_i = R^T(X) b \tag{4}$$

where $R^T(X)$ is the radial basis function and b is the coefficient vector.

The matrix form of $R(X)$ is as follows:

$$R = \begin{bmatrix} R_1(x_1, y_1) & R_2(x_1, y_1) & R_3(x_1, y_1) & \cdots & R_{10}(x_1, y_1) \\ R_1(x_2, y_2) & R_2(x_2, y_2) & R_3(x_2, y_2) & \cdots & R_{10}(x_2, y_2) \\ R_1(x_3, y_3) & R_2(x_3, y_3) & R_3(x_3, y_3) & \cdots & R_{10}(x_3, y_3) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_1(x_{10}, y_{10}) & R_2(x_{10}, y_{10}) & R_3(x_{10}, y_{10}) & \cdots & R_{10}(x_{10}, y_{10}) \end{bmatrix} \tag{5}$$

The error is calculated by the absolute value of the difference between the calculated value and the theoretical value.

Compared the interpolation results with radial basis function and accumulative point method, the radial basis interpolation method does not deviate the geological body on the whole, and can better interpolate the target body, even cute fitting shape. As high precision are the key elements of geophysical inversion, radial basis interpolation method can also meet the constraints of physical conditions, that is non-negative, so it can show better numerical calculation and geophysical inversion. The overall error can be controlled within a relatively small range; Because of the shape function, there is no serious boundary mutation problem. However, there are some shortcomings. The local extreme value with large error is near the abnormal body of the model.

In order to better display the results and prepare for the next step of inversion weight update, we present and analyze the surface model data and the results obtained by the two methods. Point interpolation has a good performance in controlling the overall shape, but there are local deviations, which will have a great impact on the accuracy of geophysical inversion. However, the accuracy of radial basis function mesh-free method is excellent in the interpolation results of the dense nodes and the middle nodes, but there are still some errors in the boundary problem. These are the next work in our efforts.

Key words: geophysical exploration, gravity forward modeling, mesh-free method, radial basis function

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