Compositions, Proportions, and Equilibrium Temperature of Coexisting Two-feldspar in Crystalline Rocks

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Abstract: Compositions, proportions, and equilibrium temperature of coexisting two-feldspar in crystalline rocks are of great importance to classification in petrography and interpretation of petrogenesis. Crystalline rocks are usually composed of 4-6 minerals (phases), depending on their independent chemical components and the equilibrium temperature of crystallizations. In general, number of mineral phases can be determined by the "Phase Rule". According to the mass balance principle, bulk composition of coexisting two-feldspar could be evaluated from the bulk chemistry of a rock, provided that the compositions of the coexisting mafic mineral phases containing calcium, sodium, and potassium oxides are determined, e.g., by microprobe analysis. The compositions, proportions, and temperature of two-feldspar in equilibrium can thus be simultaneously resolved numerically from bulk composition of the rock, by incorporating the activity/composition relations of the ternary feldspars with the mass balance constraints. Upon the numerical approximation method presented in this paper, better-quality, internally consistent data on feldspar group could usually be obtained, which would be expected more realistic and accurate in consideration of thermodynamic equilibria in the system of crystalline rocks, as well as bulk chemistry of a rock and the composing minerals.

Key words: crystalline rocks, mass balance principle, feldspar proportion, activity equation, equilibrium temperature

1 Introduction

Feldspars are the most common rock-forming minerals in crystalline rocks. The compositions and proportions of coexisting two-feldspar are the bases for classification and nomenclature of the rocks, and the crystallization temperature determined from the two-feldspar thermometer is important for petrogenetic implications. This is the reason why accurate methods of determining the compositions and proportions of two-feldspar in crystalline rocks have eagerly been sought for more than a century. Examples include the famous CIPW norm scheme by pioneer Cross et al. (1902), the description of the stable mineral assemblages of igneous rocks by Rittman (1973), conversion algorithm of chemical compositions to their natural minerals in granitic rocks (Zhu Weifang, 1979), the idealized mineral amounts in volcanic lavas (Wu Liren et al., 1982), which incorporated the advantages of simplicity of the CIPW norms by Cross et al. (1902) and accurate assigning three end members of An, Ab, and Or into the

two series of alkali-feldspar and plagioclase by Rittman (1973), and so on.

Nevertheless, such norms or stable minerals are frequently in disagreement with the modal compositions of two coexisting feldspars, and more realistic and rigorous approaches to resolve this tricky question are still of great significance to advanced petrologic researches, such as "petrographic mixing equation" approximation by the least-squares fitting and the linear programming methods (Bryan et al., 1969; Lin Wenwei, 1987). More recently, Fuhrman et al. (1988) analyzed the common expected microprobe analytical errors (±2 mol%) and the alkali exchange between and within feldspar crystals after crystallization, and developed a revised thermodynamic model for ternary feldspars for the two-feldspar geothermometer. This thermometer can give further information on how the feldspar compositions may have been changed by post-crystallization processes such as alkali exchange and subsolidus exsolution. Kroll et al. (1993) further analyzed the difference between the (Na, K)-Ca (Al-Si) and Na-K exchanges of two feldspars, and revised the model applicable for slowly cooled crystalline

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rocks. One of the most practical applications of this model and thermometer is as a test for equilibrium between co-occurring feldspars (Fuhrman and Lindsley, 1988), thus allowing the equilibrated compositions of coexisting two feldspars to be recovered in practical uses (Hokada, 2001).

On the basis of analysis of the principal composing components of common crystalline rocks, an algorithm has been developed in this study to simultaneously resolve compositions, proportions, and temperature of two-feldspar in equilibrium from bulk composition of a rock, by combining the mass balance principle with thermodynamic constraints, provided known compositions of the coexisting mafic minerals containing calcium, sodium, and potassium oxides, which are often more homogeneous in micro-structure than feldspars (Kroll et al., 1993), and easy to be determined, e.g., by microprobe analysis in practice.

2 Basic Principles

The constituent components of common crystalline rocks are typically SiO_2 (Al_2O_3), Al_2O_3 (Fe_2O_3, Cr_2O_3), FeO (MgO, NiO, MnO, TiO_2), CaO (Na_2O), K_2O (Na_2O), P_2O_5, and H_2O (F), and each cation in the oxides could be substituted in common mineral structures by those listed in the parentheses afterward. The intensive variables to govern the equilibria are only temperature and pressure in most cases, and both can vary independently over a range, i.e., degrees of freedom $f = 2$. From the well-known Goldschmidt's mineral phase rule developed in 1918 (Goldschmidt, 1954), it can be written that:

$$\phi \leq n \tag{1}$$

Where ϕ is the number of mineral phases, and n, the number of components. When H_2O in the system is present as a fluid phase to influence the equilibrium, then $n = 6$, $\phi = 6$ or 5 (while pressure equals to P_{H_2O}).

The mineral assemblages occurring in most crystalline rocks are known as (Hughes, 1982):

(1) *alkaline gabbros*: clinopyroxene, biotite, plagioclase, alkali feldspar, ferrotitanium oxide, and apatite.

(2) *monzonites and diorites*: hornblende, clinopyroxene or biotite, plagioclase, alkali feldspar, quartz, ferrotitanium oxide, and apatite.

(3) *granites*: hornblende, biotite or muscovite, plagioclase, alkali feldspar, quartz, ferrotitanium oxide, and apatite.

Of these minerals, quartz, ferrotitanium oxide (magnetite, ulvospinel, ilmenite, rutile), and apatite are often close to stoichiometry in composition. The constituent components of two-feldspar include SiO_2, Al_2O_3, CaO, Na_2O, and K_2O. Compositions of feldspars can readily be expressed in terms of the end member An ($CaAl_2Si_2O_8$), Ab ($NaAlSi_3O_8$), and

Or ($KAlSi_3O_8$), and seldom depart from stoichiometry significantly, although $BaAl_2Si_2O_8$, $SrAl_2Si_2O_8$, even $CaMgSi_3O_8$, $CaFeSi_3O_8$, Si_4O_8 could rarely be incorporated in minor concentrations (Longhi, 1976; Ma Hongwen, 1990; 2001). Hence the bulk composition of two-feldspar can be derived from chemical composition of the rock, provided that the compositions of the one or two coexisting mafic mineral phases are known.

Equilibrium experiments on model granite to granodiorite systems show that the solidus mineral is biotite in low H_2O content at low pressure (p_{H_2O}=0.2 GPa); biotite or epidote at intermediate water pressure (p_{H_2O}=0.8 GPa); quartz or alkali feldspar in high H_2O content; whereas solidus minerals are epidote, plagioclase or alkali feldspar at intermediate pressure (Naney, 1983).

The residual silicate liquids of alkaline basaltic, intermediate to granitic magmas would reach or close to their eutectic compositions, regardless of the crystallization path of the magma, i.e., occurring equilibrium crystallization of two-feldspar, and the most possible mineral assemblage would be plagioclase + alkali feldspar ± quartz, as outlined by the famous Bowen's reaction series (Bowen, 1922, 1928), and experimentally demonstrated by Piwinskii (1973) for calc-alkaline rock series. In case of what the bulk composition of two-feldspar is resolved numerically, the compositions, proportions and temperature of the coexisting two-feldspar in equilibrium in a magma with the eutectic composition could therefore be evaluated simultaneously on the basis of the activity/composition model of the ternary feldspars (Ghiorso, 1984; Fuhrman and Lindsley, 1988; Elkins and Grove, 1990), and more recently the ternary mixing model for high structural state feldspars (Benisek et al., 2009; 2010a; 2010b).

3 Bulk Composition of Coexisting Two-feldspar

Suppose that a crystalline rock is composed of m chemical components and n minerals, the sum of products of each mineral proportion x_j by the component a_{ij}, is thus equal to the component of the rock b_i based on the mass-balance principle (Ma Hongwen, 2001), i.e.,

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \ (i = 1, 2, \ldots\ldots, m) \tag{2}$$

The objective of evaluating mineral proportions is to assign the m components to the n minerals, giving the minimum squares of the residual, i.e., the maximum sum of the n proportions (close to 1.0). In mathematic terms, this is a linear programming problem to solve for the maximum value of $S = \sum_{j=1}^{n} x_j$, which gives $AX = B$, and $X \geq 0$ (Ma Hongwen, 2001). The result can then be evaluated by the simple linear programming method (Lin Wenwei, 1987) or the least-squares algorithm (Bryan et al., 1969).

In the latter case, the estimated mineral proportions could be negative while unreasonable mineral phases or unstable mineral compositions are incorporated in the calculations (Lin Wenwei, 1987). To determine the bulk composition of two-feldspar conveniently from composition of a crystalline rock, the ternary compositions of the end member An, Ab, and Or (in mole fractions) should be adopted in the calculation.

4 Compositions, Proportions, and Temperature of Two-feldspar in Equilibrium

The point to evaluate the compositions of two-feldspar is to assign the ternary end members to the coexisting two-feldspar phases. When two feldspars in a rock are in equilibrium, then

$$\mu_i^{pl} = \mu_i^{af} \quad (i = \text{An, Ab, Or}) \tag{3}$$

Where pl and af are plagioclase and alkali feldspar, respectively, and the chemical potentials μ_i of the end member i, in either plagioclase or alkali feldspar phase, is described by the free energy in the standard state and the mixing free energy, which are given by:

$$\mu_i^{pl} = \mu_i^0 + RT\ln a_i^{pl} \tag{4}$$
$$\mu_i^{af} = \mu_i^0 + RT\ln a_i^{af} \tag{5}$$

In the case that plagioclase and alkali feldspar are regarded as two ternary solid solutions, then the activity of each end-member in two-feldspar is linked by the same free energy term in the standard state. As μ_i^0 in both feldspar phases are equal, the equilibrium is then simply described as:

$$RT\ln a_1^{pl} = RT\ln a_i^{af} \quad (i = \text{An, Ab, Or}) \tag{6}$$

Adapting the activity/composition relations in the ternary feldspars (Fuhrman and Lindsley, 1988; Elkins and Grove, 1990; Benisek et al., 2010b) gives:

$$
\begin{aligned}
a_{Ab} = x_{Ab} \times \exp\{&(W_{OrAb}[2x_{Ab}x_{Or}(1-x_{Ab}) + x_{Or}x_{An}(1/2-x_{Ab})] \\
&+ W_{AbOr}[x_{Or}^2(1-2x_{Ab}) + x_{Or}x_{An}(1/2-x_{Ab})] \\
&+ W_{AbAn}[x_{An}^2(1-2x_{Ab}) + x_{Or}x_{An}(1/2-x_{Ab})] \\
&+ W_{AnAb}[2x_{An}x_{Ab}(1-x_{Ab}) + x_{Or}x_{An}(1/2-x_{Ab})] \\
&+ W_{OrAn}[x_{Or}x_{An}(1/2-x_{Ab}-2x_{An})] \\
&+ W_{AnOr}[x_{Or}x_{An}(1/2-x_{Ab}-2x_{Or})] \\
&+ W_{OrAbAn}[x_{Or}x_{An}(1-2x_{Ab})])/RT\}
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
a_{An} = x_{An} \times \exp\{&(W_{OrAb}[x_{Ab}x_{Or}(1/2-x_{An}-2x_{Ab})] \\
&+ W_{AbOr}[x_{Ab}x_{Or}(1/2-x_{An}-2x_{Or})] \\
&+ W_{AbAn}[2x_{Ab}x_{An}(1-x_{An}) + x_{Ab}x_{Or}(1/2-x_{An})] \\
&+ W_{AnAb}[x_{Ab}^2(1-2x_{An}) + x_{Ab}x_{Or}(1/2-x_{An})] \\
&+ W_{OrAn}[2x_{Or}x_{An}(1-x_{An}) + x_{Ab}x_{Or}(1/2-x_{An})] \\
&+ W_{AnOr}[x_{Or}^2(1-2x_{An}) + x_{Ab}x_{Or}(1/2-x_{An})] \\
&+ W_{OrAbAn}[x_{Or}x_{Ab}(1-2x_{An})])/RT\}
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
a_{Or} = x_{Or} \times \exp\{&(W_{OrAb}[x_{Ab}^2(1-2x_{Or}) + x_{Ab}x_{An}(1/2-x_{Or})] \\
&+ W_{AbOr}[2x_{Ab}x_{Or}(1-x_{Or}) + x_{Ab}x_{An}(1/2-x_{Or})] \\
&+ W_{AbAn}[x_{Ab}x_{An}(1/2-x_{Or}-2x_{An})] \\
&+ W_{AnAb}[x_{Ab}x_{An}(1/2-x_{Or}-2x_{Ab})]
\end{aligned}
$$

$$
\begin{aligned}
&+ W_{OrAn}[x_{An}^2(1-2x_{Or}) + x_{Ab}x_{An}(1/2-x_{Or})] \\
&+ W_{AnOr}[2x_{Or}x_{An}(1-x_{Or}) + x_{Ab}x_{An}(1/2-x_{Or})] \\
&+ W_{OrAbAn}[x_{An}x_{Ab}(1-2x_{Or})])/RT\}
\end{aligned}
\tag{9}
$$

Where x_i is the mole fraction of end member i, and W terms are the excess free energy parameters for ternary feldspars (Elkins and Grove, 1990; Table 1).

To describe the compositions, proportions, and equilibrium temperature of two-feldspar at a specific pressure, the following equations are given:

$$a_i^{pl} = a_i^{af} \qquad (i = \text{An, Ab, Or}) \tag{10}$$
$$x_{An}^j + x_{Ab}^j + x_{Or}^j = 1.0 \qquad (j = \text{rck, pl, af}) \tag{11}$$
$$y_1 x_i^{pl} + y_2 x_i^{af} = x_i^{rck} \qquad (i = \text{An, Ab, Or}) \tag{12}$$
$$y_1 + y_2 = 1.0 \tag{13}$$

Where the superscript rck, pl, and af denote the whole rock, plagioclase, and alkali feldspar, respectively. The terms y_1 and y_2 are the proportions of the two-feldspar phases. In the above equations, x_{An}^{rck}, x_{Ab}^{rck}, x_{Or}^{rck} in the rock are known variables, and the relevant pressure P can be estimated using geobarometry methods (Ma Hongwen, 2001).

From the above, there are ten unknowns in the above ten questions, i.e., the mole fractions of three end members (x_{An}^{pl}, x_{Ab}^{pl}, x_{Or}^{pl}) in plagioclase, the mole fractions of three end members (x_{An}^{af}, x_{Ab}^{af}, x_{Or}^{af}) in the alkali feldspar, the three temperatures (T_{An}, T_{Ab}, T_{Or}) obtained from Eqs. 7-9, and either of the proportions y_1 or y_2. Thus the number of variables to be resolved equals to the number of equations given above. Mathematically, the equations could be simultaneously resolved with a unique solution.

5 Discussions and Applications

In igneous rocks, alkali feldspar is much more variable than pyrogenetic plagioclase, and usually comprises optically homogeneous sanidine and anorthoclase in young volcanic rocks, cryptoperthites in old volcanic rocks, and orthoclase and microcline frequently perthitic in plutonic rocks. In granitic rocks, pronounced exsolution and/or late-stage recrystallization and alteration have resulted in the formation of nearly pure albite in composition, which may be more intimately mixed with potassium feldspar in perthites (Hughes, 1982). As the main phase present in alkali lavas, the abundance of alkali feldspar can vary strongly with small changes in pressure,

Table 1 The excess free energy terms for ternary feldspars

	W_H	W_S	W_V
W_{AbOr}	18810	10.3	4602
W_{OrAb}	27320	10.3	3264
W_{AbAn}	7924		
W_{AnAb}	0		
W_{OrAn}	40317		
W_{AnOr}	38974		−1037
W_{OrAbAn}	12545		−10950

Note: (1) $W_G = W_H - TW_S + pW_V$; (2) Units as J(W)、K(T)、GPa(p).

temperature and water content in the melt, implying appreciable variations in crystallization kinetics and the resulting textures (Arzilli et al., 2013). In granites, K-feldspar is typically the last major phase to crystallize and that most K-feldspar growth occurs after the magma crosses the rheologic lock-up threshold of ~50% crystals. The highly potassic compositions of megacrysts of K-feldspar in granites require exsolution of albite component down to temperatures of ~400℃. The low Ca contents of megacrysts cannot result from exsolution of anorthite and must represent recrystallization of the crystals at low temperature (Glazner et al., 2013).

In high grade metamorphic rocks slowly cooled under essentially dry conditions, meanwhile, non-equilibrium compositions of coexisting feldspars result usually from retrograde intercrystalline K-Na exchange. This K-Na exchange continues after two feldspars have already become essentially closed systems with respect to Al-Si exchange (Kroll et al., 1993). However, essentially "dry" high-pressure felsic granulites can contain perthitic alkali feldspars up to 2 mm in size with several generations of plagioclase precipitates in an orthoclase-rich host (Tajčmanová et al., 2012).

In Paleoproterozoic granitoids, formation of flame perthite can be attributed to replacement reaction mechanism (Na-K exchange) between K-feldspar and plagioclase under low to moderate differential stress condition during rapid cooling. Further, extensive deuteric alteration results in the formation of braid and vein perthites (Pandit, 2015).

Feldspar crystals typically show a range of exsolution and polysynthetic twinning textures that can present problems for their full characterization, but at the same time give important information about their genesis (Balić-Žunić et al., 2013). As a result, the true compositions of two-feldspar in equilibrium could seldom be precisely determined even by microprobe analysis.

As an example, compositions, proportions, and equilibrium temperatures of the coexisting two-feldspar in the Badaling plutons, north of Beijing, China, have been calculated following the schedule outlined above from the bulk compositions of the granites (Table 2). The calculated phase diagram of the equilibrated two feldspars in the granites is shown in Fig. 1.

A modified thermodynamic algorithm was used to reverse both of the K-Na and Al-Si exchanges (Kroll et al., 1993) and derive concordant temperatures (Fuhrman and Lindsley, 1988; Elkins and Grove, 1990). It is worth noting that use of the least-squares algorithm (Bryan et al., 1969) may lead to negative proportions, possibly caused by unreasonable minerals or non-equilibrium compositions being adopted in the calculation (Lin Wenwei, 1987). For high structural state feldspars, such as those in trachytic to rhyolitic lavas and high-grade metamorphic rocks, the newly developed mixing model and the interaction parameters for ternary feldspars in the Ab-Or-An system should be used (Benisek et al., 2009; 2010a; 2010b).

The temperatures derived in this way represent the closure temperatures for the intercrystalline Al-Si exchange (Kroll et al., 1993). These temperatures are higher than the solidus of granite to granodiorite magmas at P_{H_2O} = 2–5 kbar, and usually alkali feldspar did not crystallize until temperatures down to 10–20℃ higher than the solidus (Piwinskii, 1973), indicating that the as-predicted temperatures could be close to the solidus, or represent the maximum temperature of exsolution. For metamorphic rocks, these temperatures represent the
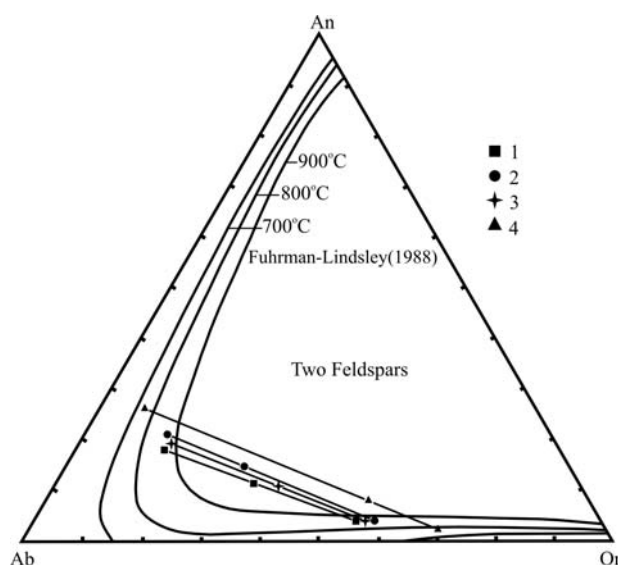

Fig. 1. Calculated phase diagram of equilibrated two feld-spars in the granitic and syenitic rocks as examples.
1-3, biotite monzogranite of the Badaling plutons (cf. Table 2), Bejing city; 4, hornblende syenite of Xianghongdian, Jinzhai county of Anhui province

Table 2 Compositions, proportions, and equilibrium temperature of the coexisting two-feldspar in the Badaling granite plutons

Plutons	Two-feldspar bulk composition			Compositions						Proportions (%)		Equilib-rium T (℃)
				Plagioclase			Alkali feldspar			Plagioclase	Alkali feldspar	
	x_{An}	x_{Ab}	x_{Or}	x_{An}	x_{Ab}	x_{Or}	x_{An}	x_{Ab}	x_{Or}			
1-Duiju	0.121	0.556	0.323	0.178	0.667	0.156	0.044	0.414	0.542	56.7	43.3	875
2-Tielu	0.146	0.547	0.307	0.207	0.652	0.141	0.045	0.382	0.573	61.5	38.5	876
3-Tiecn	0.117	0.519	0.364	0.194	0.654	0.152	0.047	0.401	0.552	47.0	53.0	883

minimal but close to the true peak temperature, as the intercrystalline Al-Si exchange is expected to cease early in the cooling history (Kroll et al., 1993).

A hornblende syenite sample (XS-13) collected from Xianghongdian of Jinzhai County, Anhui Province, China (Zeng et al., 2015; Yang Jing et al., 2016), is used as an example of how the algorithm can be applied in practice. The common procedure involves three steps, as described below.

(1) Determine the bulk composition of the hornblende syenite (XS-13) by wet chemical or X-ray fluorescence analysis, and chemical compositions of the major minerals by microprobe analysis (Table 3).

(2) Determine mineral proportions of the hornblende syenite (XS-13) in weight percent using the simple linear programming algorithm (Lin Wenwei, 1987), which is performed by program LINPRO.F90 (Ma Hongwen, 1999). The ideal compositions of end members of An, Ab, and Or of the two feldspars are used in the calculation, which yields the following proportions: Or 50.81%, Ab 33.19%, An 6.76%, hornblende 5.37%, magnetite 2.07%, and sphene 1.00%. The approached objective function is 0.9930 which, considering the uncertainties of different analytical methods and neglecting minor mineral phases (e.g., apatite and zircon) in the calculation, is a reasonable value.

(3) Estimate compositions, proportions, and equilibrium temperature of coexisting two-feldspar in the hornblende syenite (XS-13) using the thermodynamic model of Fuhrman and Lindsley (1988) and the least-squares algorithm of Bryan et al. (1969) calculated using the program FLDCRY.F90 (Ma Hongwen, 1999). The approximated compositions and proportions of the target feldspars are: plagioclase, $An_{0.257}Ab_{0.668}Or_{0.075}$, 23.0wt%; K-feldspar, $Or_{0.692}Ab_{0.286}An_{0.022}$, 77.0wt%; and the two feldspars equilibrated at approximately 762.6℃ (T_{An}=762.2℃, T_{Ab}=761.9℃, T_{Or}=763.6℃).

The LINPRO·F90 and FLDCRY·F90 program source codes used in the calculations are available from the first author upon request.

For comparison, the realistic compositions of the two feldspars determined by microprobe analysis are listed in Table 3, which gives the end members as: plagioclase, $An_{0.262}Ab_{0.728}Or_{0.010}$; and K-feldspar, $Or_{0.722}Ab_{0.264}An_{0.014}$.

The absolute differences of these compositions from the microprobe analyzed values are therefore only An 0.005, Ab 0.060, Or 0.065 for plagioclase; and An 0.008, Ab 0.022, Or 0.030 for K-feldspar, respectively.

It is evidenced that the uncertainties of microprobe analysis are usually around ±0.02 molar fractions of the end members for feldspars (Fuhrman and Lindsley, 1988). Taking into consideration of this analytical uncertainty and the error accumulation of different analytical methods as well as possible lamellar exsolution or compositional zoning textures of feldspars (Hokada, 2001; Kroll et al., 1993; Parsons et al., 2015; Flude et al., 2012), the estimated compositions and proportions of the two feldspars in the syenite are considered to be accurate.

As a result, this approach gives mineral proportions of the syenite as (wt%): K-feldspar (microcline) 69.9, plagioclase (oligoclase) 20.9, hornblende 5.4, magnetite 2.1, and sphene 1.0.

Almost all alkali feldspar crystals contain a rich inventory of exsolution, twin, and domain microtextures that form subsequent to crystal growth and provide a record of the thermal history of the crystal and often of its involvement in replacement reactions (Parsons et al., 2015). Although meaningful results can be obtained using the method developed in this paper, we strongly emphasize the necessity that using an integrated procedure for the microtexture analysis, twin law identification plus crystal structure refinement of all components in a feldspar intergrowth. This procedure should be applied to perthitic intergrowths in feldspars whenever possible (Balić-Žunić et al., 2013), and charge contrast imaging in the scanning electron microscope can provide new insights into the scale and composition of alkali feldspar microtextures (Flude et al., 2012). The electron-microprobe mapping method can also be used to investigate plagioclase compositional evolution due to cooling kinetics (Iezzi et al., 2014). Such information also helps considerably with the interpretation of the geological history of a rock.

In addition to being a common rock-forming mineral, K-feldspar is also a principal potassium silicate mineral as insoluble potassium resource (Ma Hongwen et al., 2010), from which various potassium salts or potash fertilizers could be easily manufactured by the hydrothermal alkaline digestion method (Ma Hongwen et al., 2014, 2015).

Table 3 Chemical compositions of hornblende syenite and the composing minerals (wt%)

Phases	SiO$_2$	TiO$_2$	Al$_2$O$_3$	Fe$_2$O$_3$	FeO	MnO	MgO	CaO	Na$_2$O	K$_2$O	P$_2$O$_5$	LOI
K-feldspar	64.75	0.02	18.31		0.17	0.01	0.01	0.31	3.18	13.20	0.05	
plagioclase	60.71	0.04	23.74		0.29	0.03	0.00	5.74	8.84	0.19	0.00	
hornblende	39.04	3.10	11.79	2.17	15.09	0.85	10.13	10.76	2.79	2.10	0.00	
sphene	30.89	35.27	2.68		2.84	0.04	0.04	26.89	0.10	0.00	0.22	
magnetite	0.00	3.14	0.85	63.64	33.29	0.00	0.11	0.00	0.00	0.00	0.00	
Hb-syenite	59.69	0.58	18.56	1.39	1.39	0.12	0.89	2.18	4.30	8.51	0.05	1.88

Note: Bulk composition of the Hb-syenite (XS-13) was analyzed by wet chemical analysis; and the mineral compositions by microprobe analysis, the ferric and ferrous oxide contents of hornblende and magnetite are calibrated by the valence equilibrium principle (Ma Hongwen, 2001).

Similarly, the large quantity of K-feldspar in mining tailings of the porphyry copper and molybdenum of granite-type deposits (Ying Jing et al., 2015; Huang Fan et al., 2015), and lithium of granite or granitic pegmatite-type deposits (Li et al., 2015) could also be an important potential potassium resource (Yu Chuanbing, 2014). As metal minerals in these deposits are usually copper and molybdenum sulfides, or lithium silicates, the proportions of two-feldspar in the metal-hosted rocks are seldom changed during the ore mineral beneficiation processes. Therefore, the method presented in this paper could be conveniently used for accurate estimatation of the proportions of the two feldspars in the mine tailings. This would provide critical input for both of designing the process scheme of the tailings (Yu Chuanbing, 2014), and evaluating the feasibility of procedural reactions using physicochemical and thermodynamic theories (Ma Hongwen et al., 2007).

6 Conclusions

(1) The compositions, proportions, and equilibrium temperature of coexisting two-feldspar in crystalline rocks can be simultaneously resolved numerically from bulk composition of the rocks, on the basis of incorporating the activity/composition relations of the ternary feldspars with mass balance constraints. The resultant information forms the basis for classification and nomenclature of the rocks, and interpretation of their petrogenesis.

(2) The exact compositions and proportions of coexisting two-feldspar are important for the design of the granite or granitic pegmatite-type metal ore tailing facilities, and evaluating the feasibility of the procedure reactions using physicochemical and thermodynamic theories.

(3) The methodology developed in this paper provides an alternative to conventional petrological methods, and is constructed on a solid foundation of thermodynamics and accurate compositional constraints using numerical algorithms. Thus, it is possible that more accurate and universally applicable results could be derived from the conventional bulk compositions of crystalline rocks and compositions of the paragenetic minerals.

Acknowledgements

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