A STATISTICAL STUDY OF THE DIFFERENCE BETWEEN THE WIDTH-HEIGHT RATIO OF *SPIRIFER TINGI* AND THAT OF *SPIRIFER HSIEHI*.

BY

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By V. K. TING (丁文江)

In my geological reconnaissance in east Yunnan in 1914 I collected from a series of shales, sandstones and impure limestones, which underly the Lower Devonian beds containing *Arthrostigma gracile*, a large number of small, but perfectly preserved spirifers, which Dr. A. W. Grabau named *Spirifer tingi* and considered to be of Silurian age. The series was named by me the Miaokao (妙高) formation from a hill outside the district city of Chūts'ing (曲端). Below it is the Mientien (麵店) formation, very similar in petrological character, but containing a slightly different fauna. Apparently neither of these formations is found in the Yangtze Gorges where the uppermost bed of Sintan shale is overlaid by the Wushan limestone. The Sintan shale, however, contains also a small spirifer, named by Dr. Grabau *Spirifer hsiehi* which is very similar to *S. tingi*. Dr. Grabau at first distinguished the two closely allied species by the character of the sinus. In *Spirifer tingi* it is plicated, whilst *S. hsiehi* shows no sinal plication. In 1930 I found in southern Kueichow the same Miaokao formation with *Spirifer tingi* underlying the Devonian, but there it overlies a series of beds containing *S. hsiehi*. Dr. Grabau regards the Miaokao formation as the uppermost Silurian but the underlying *S. hsiehi* beds as equivalent to the Sintan shale. which is considered to be lower Silurian.

In 1931 Mr. Y. L. Wang revisited my locality in Chūts'ing and made a much larger collection of *S. tingi*. Moreover, he found that *S. tingi* occurs also in a lower horizon of the Mientien formation, separated from the Miaokao *S. tingi* beds by several hundred meters of sediments. Amongst his specimens, which number more than 900, a fairly large number developed no sinal plication. Thus this character can no longer be used as the means of distinguishing the two species. Dr. Grabau, however, still regards them as separate species because the ratio of the width to the height in *S. hsiehi* seems to be much greater than that in *S. tingi*, both in the plicated and the non-plicated forms. He further separates *S. tingi* into several varieties. As the two species occur in

quite different formations and are extremely abundant, they would serve as excellent index-fossils if we could be sure of their specific difference.

It seems to me that it is not enough to know that the mean ratio of width to height is different in the two species, but we must determine whether the difference is *statistically significant*. It is well-known that if the mean of two series of measurements be M and M', and their probable error be E and E', then the probable error of M—M' will be $\sqrt{E^2+E'^2}$. The quantity $\dfrac{M-M'}{\sqrt{E^2+E'^2}}$ is known as the *significant difference*. If it is greater than three, it is usually considered to be significant. On the other hand, if the result is less than three, the apparent difference may be due to sampling, and the two objects measured may be variations of a single series.

At my suggestion Dr. Grabau made the following series of measurements of the width and height of both the pedicle and the brachial valve :

TABLE I.

Series	Species	Locality	Number of Specimens Pedicle Valve	Brachial Valve
a	S. *tingi* var. *simplex* (sinus non-plicated)	Y33	50	50
b	S. *tingi* var. *typica* (sinus plicated)	Y33	50	50
e	S. *tingi* var. *antecedens* (sinus plicated)	Y45	40	40
d	S. *tingi* var. *antecedens* mut. β (sinus non-plicated)	Y45	34	30
e	S. *sublingi* (sinus non-plicated)	Y189	50	50
f	S. *hsiehi*	T11, T332, Tw56 etc.	32	16

From his measurements I have calculated the mean (M), the standard deviation (σ) and the probable error (E) of each series as follows[*]:—

TABLE II. Width-Height Ratio $\left(\dfrac{W}{H} \times 100\right)$ of the pedicle valve.

Series	M	σ	E
a	113.52	6.24	0.60
b	115.12	6.27	0.60
c	116.98	6.63	0.71
d	116.71	9.16	1.06
e	121.03	8.40	0.80
f	147.32	11.80	1.41

TABLE III. Width-Height Ratio $\left(\dfrac{W}{H} \times 100\right)$ of the brachial valve.

Series	M	σ	E
a	139.95	9.50	0.91
b	138.77	8.07	0.77
c	140.67	8.85	0.94
d	138.30	7.26	0.89
e	140.89	7.44	0.71
f	159.05	12.20	2.06

From these tables the quantity $\dfrac{M - M'}{\sqrt{E^2 + E'^2}}$ was calculated.

The mean (M) is the average of all the proportions in each series, obtained by dividing the total of the proportions by the number of individuals. The standard deviation (σ) is the square root of the mean square deviation from the mean, obtained by the formula $\sqrt{\dfrac{S\,(M-m)^2}{n}}$ where m represents the individual measurements, M the mean, n the number of individuals and S summation. The probable error (E) is obtained by the formula $0.6745 \times \dfrac{\sigma}{\sqrt{n}}$.

TABLE IV. $\dfrac{M - M'}{\sqrt{E^2 + E'^2}}$ for the pedicle valve.

Series	a	b	c	d	e	f
a	——	1.88	3.72	2.61	7.51	22.09
b	1.88	——	2.00	1.30	5.91	21.05
c	3.72	2.00	——	0.21	3.79	19.20
d	2.61	1.30	0.21	——	3.25	17.39
e	7.51	5.91	3.79	3.25	——	16.23
f	22.09	21.05	19.20	17.30	16.23	——

TABLE V. $\dfrac{M - M'}{\sqrt{E^2 + E'^2}}$ for the brachial valve.

Series	a	b	c	d	e	f
a	——	0.99	0.55	1.30	0.82	8.49
b	0.99	——	1.57	0.40	2.92	9.22
c	0.55	1.57	——	1.84	0.19	8.04
d	1.30	0.40	1.84	——	2.27	9.26
e	0.82	2.02	0.19	2.27	——	8.29
f	8.49	9.22	8.04	9.26	8.29	——

From the above it is clear that as far as the character of width to height ratio is concerned, *Spirifer hsiehi* differs significantly for *S. tingi* whether the latter shows sinal plication or not, for the quantity $\dfrac{M - M'}{\sqrt{E^2 + E'^2}}$ varies between 16 and 22 in the pedicle valve and between 8 and 9 in the brachial valve, when *S. hsiehi* (series f) is compared with the other five series of *S. tingi*. On the other hand, the value for the latter species is ever so much smaller when they are compared by themselves.

In the latter case the results are of unusual interest. For series a and b, which are both from horizon Y33, the value of $\dfrac{M - M'}{\sqrt{E^2 + E'^2}}$ is 0.99 for the brachial, and 1.88 for the pedicle valve. These figures indicate clearly that the two series are practically identical, although b has a plicated sinus and a has not. Similarly for series c and d, both of which come from horizon Y45, the value of $\dfrac{M - M'}{\sqrt{E^2 + E'^2}}$ is less than one in the pedicle valve in spite of their difference in sinal plication.

For the brachial valve, the value of $\frac{M-M'}{\sqrt{E^2+E'^2}}$ for all the five series a-e, when each is compared with the other, is all under 2, except that for series b and e which reaches 2.02. and d and e where it reaches 2.27. Consequently, as far as the brachial valve is concerned, the five series may be considered identical. But it is not so with the pedicle valve. Here the value of $\frac{M-M'}{\sqrt{E^2+E'^2}}$ for the series a, b, c and d, when compared with themselves, is all less than 3, except that between a and c which reaches 3.72. On the other hand, when the series a, b, c and d are compared with e, the value of $\frac{M-M'}{\sqrt{E^2+E'^2}}$ is all above 3—3.25 for c, 3.79 for d, 5.91 for a, and 7.51 for b. Here the difference is undoubtedly significant.

Moreover, Table IV shows clearly that series a and b differ most from series f, series c and d somewhat less, and the difference between series e and f is far less marked.

These conclusions arrived at from purely statistical considerations agree very well with the stratigraphical evidence, for the vertical distribution of these series is as follows (in the descending order):—

Miaokao Formation I. Y33 (series a and b)

II. Y45 (series c and d)

Mientien Formation   Y189 (series e)

Sintan Shale   T11 etc. (series f)

In fact horizon Y189 is separated from Y33 by more than 500 m of sediments, and Y45 lies definitely below Y33. If we assume that *Spirifer hsiehi* is the ancestral form of *S. tingi* we would expect series e to be the nearest to series f, series c and d next, and series a and b the farthest in specialisation and differentiation. This is precisely the result we arrived at from our statistical studies.

It is to be noted that horizon Y189 (series e) contains only forms with a non-plicated sinus. Sinal plication therefore must be a later specialisation. This seems to have no correlation with the differentiation of the width-height ratio. For specimens from the same horizon have always a very similar width-height ratio irrespective of the presence or absence of sinal plication. Thus

the value of $\dfrac{M - M'}{\sqrt{E^2 + E'^2}}$ for comparison between a and b (both from horizon

Y33) is 1.88, and that between c and d (both from Y45) is 0.21, although b and c have a plicated sinus and a and d have not. On the other hand, when specimens from different horizons are compared, the result shows no relation to sinal plication. Series e (with non-plicated sinus) from horizon Y189 is not closer to d (sinus non-plicated) than to c (sinus plicated) from the same horizon Y33. In fact, series e shows actually a greater significant difference when compared with the non-plicated specimens from a than when compared with the plicated forms from b, the value being 7.51 for the former and 5.91 for the latter. Thus sinal plication and decrease of the width-height ratio are two characters that have been evolved independently without any correlation.

We may consider that *Spirifer hsiehi* and *S. tingi* belong to different species. They can readily be distinguished by their width-height ratio (expressed as a percentage), especially that of the pedicle valve. If this lies within 147 ± 12 (M of series f $\pm \sigma$), the specimen may be safely considered as *S. hsiehi* the sinus of which is, moreover, never plicated.

Dr. Grabau is perhaps justified in making series e a subspecies and calling it *S. subtingi*, since it differs significantly in its width-height ratio from all the other varieties of *S. tingi* (series a, b, c and d). In practice it will be less easy to distinguish *S. subtingi* from *S. tingi*. If there is a considerable number of individuals in any collection of *S. tingi* whose width-height ratio of the pedicle valve is greater than 121, we may suspect that they may have come from the Mientien Formation and may in reality belong to *S. subtingi*. But it will be difficult to determine to which of the two subspecies the individual belongs when its width-height ratio lies between 117 and 121, for here it is within the range of variation of both subspecies.

On the evidence of width-height ratio alone, it is, in my opinion, hardly justifiable to make series c and d a different variety from b and a, since the difference is not really significant. Practically it will be also very difficult to distinguish them by measurement, as the range of variation largely overlaps in the two supposed varieties.