Discussion on Geodynamics of Three-body Motion

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Abstract: To determine the Earth’s dynamics and their equations, which are crucial for Earth science research, this paper analyzes the interaction forces in the motion of a three-body system (namely, fixed, active, and passive points), based on the orbital motion. The mathematical derivation has been conducted strictly according to trigonometric functions with time and space as variables. In spatial transformation, related data items are simplified and replaced reasonably and necessarily according to the physical phenomenon to conduct derivations of planar to spatial transformation, through which the motion point has universal significance. Moreover, the polynomial equation for the dynamics has been obtained. Results indicate that the polynomial expression for the dynamics comprises the tidal force, the powerful mid-latitude Force (PML Force), and gravitation. Gravitation analysis shows that it is proportional to the dynamics quality, the size of the angular velocity of their deviation from the progenitor—paternal orbital plane’s center position, and the square of the progenitor orbital plane’s distance. However, it is inversely proportional to the distance of the paternal orbital plane and not related to another body’s quality. Some past errors are addressed and some constructive conclusions are offered in the discussion of gravitation.

Key words: Dynamics equation, universal gravitation, tidal force, powerful mid-latitude force, gravitation

1 Introduction

The motion of three bodies, as discussed by Newton, is actually the motion of two bodies moving around the third body. These bodies have a similar relationship to that of the Earth, Venus and the Sun’s motion (Newton, 1729; Chernous'ko, 1967; Enosh and Kotevitz, 1973; Yen and Huang, 1983; Chandrasekhar, 1995; Tesio and Detrembleur, 1998; Gucciardi, 1999; Vil'ke and Shatina, 2000; Rozenblat, 2011). Herein, the motion of three bodies focuses on the spinning body itself, its satellite, and the orbital revolution foci (Liu et al., 2000a, 2000b; 2001a, 2001b, 2001c, 2001d, 2001e), such as the Earth (the spinning body), the Moon (its satellite), and the Sun (the orbital revolution foci). The mechanics of these two kinds of motion relationships between three bodies differ from each other, while the key point to be probed is the same—the dynamic equations.

The dynamics among three bodies, as unraveled by Newton, are dominated by gravitation. Newton believed that gravitation could occur in any two of three individual bodies, such as the Sun and Earth, or Venus and Earth, gravitation is actually related to body quality and distance but not the orbit, and the tide on any planet surface gravitates toward the two adjacent celestial bodies.

The results herein shows that gravitation is related to the orbital foci, which results in the gravitation, and the tidal forces and the deflection force at the same time and with the same magnitude level. However, the tide is not caused by gravitation.

2 Analysis Systems

The moon revolves around the Earth’s orbital plane, or the rotational surface of one body in the Earth sphere differs from the Earth’s orbital plane. We begin at a predetermined point with a certain mass at a certain time and observe its interaction with the Earth and Sun with respect to planar motion. A specified point will then have
universal significance after its spatial transformation such that the spatial interaction situation for any motion point on the Earth can be finally constructed.

2.1 Planar analysis system

The planar analysis for the Earth's motion point (M), the Earth (E), and the Sun (S) is constructed as shown in Fig. 1 (Liu et al., 2001c, 2001d).

![Fig. 1. Planar interaction analysis within the three-body motion.](image)

In \( \Delta EMS \), setting \( \angle EMS = \theta \), \( EM = R \), \( SM = L \), \( ES = l \). According to the cosine theorem, the implicit function relation in the \( \Delta EMS \) is expressed as follows:

\[
R^2 = L^2 + l^2 - 2Ll \cos \theta
\]

(1)

where \( R \), \( L \), and \( l \) are the functions of angle \( \theta \) and time \( t \), and the relationships between them are shown as follows:

\[
R = f_1(\theta, t)
\]

\[
L = f_2(\theta, t)
\]

\[
l = f_3(\theta, t)
\]

First, we analyze the formula (1) with the derivative to determine \( \theta \), then we analyze time, introduce the angular velocity of revolution \( w \), and multiply \( m \) on both sides of the equation. Subsequently

\[
m \sqrt{u} \frac{d^2R}{dt^2} = m \frac{dA}{dt} - m \frac{1}{u} A^2
\]

(2)

where

\[
\sqrt{u} = R
\]

\[
A = l + \frac{1}{2} L \cos \theta - L \cos \theta L + l L \cos \sin \theta
\]

2.2 Spatial analysis system

Since \( R \), \( L \), and \( l \) are not coplanar, the analytical result does not have universal significance when using planar analysis; thus, spatial transformation is necessary.

Taking any point \( P \) on the spherical surface of the Earth, we develop the spatial coordinate's analysis system as point \( P \) rotates around the Earth (Fig. 2).

In Fig. 2, \( M \) is the projected point of \( P \) on the ecliptic plane and the \( \Delta EMS \) is the same as shown in Fig. 1. Correspondingly, \( \angle MSE = \beta \) and \( \angle PEM = \alpha \), which is also the ecliptic latitude and is related with the Earth latitude \( \varphi \). The equation \( \angle PSM = \gamma \) is the visual deflection angle between \( P \) and the Sun when they are away from the ecliptic plane. The equation \( \angle MES = \beta \) is the ecliptic longitude, which is the difference between the longitude of its position at noon (or midnight). As such, we obtain the following,

\[
R' = R \cos \alpha, L' = L \cos \gamma.
\]

2.3 Dynamic equations for three-body motion

Because the distance between the Earth and the Sun is vast, the magnitude of \( \gamma \) is minimal and approaches 0, such that we set \( \gamma = 0 \). Therefore, the following expression can be deduced easily:

\[
\begin{align*}
L' &= L \\ R \cos \alpha \cos \beta &= l \cos \theta \\ R \cos \alpha \sin \beta &= L \sin \theta
\end{align*}
\]

Considering \( L = l \) and setting \( L = l \), we simplify the above expression and obtain the following equation

\[
F_1 = ma = 2m \cos \alpha \cos \beta \left( l + \frac{1}{4} mR \cos^2 \gamma \right) + \frac{mL}{R} \cos \theta \left( \frac{d \theta}{dt} \right)^2
\]

(3)

or \( F_1 = m \frac{d^2R}{dt^2} \)

where \( \alpha \) is the second derivative of the distance between \( P \) and the Earth's axis in expression (3), which changes over time, and is termed the radial acceleration variation.

Expression (3) is actually the complete formula for an active force of a motion point in the Earth system as the Earth revolves around the rotating Sun (Liu et al., 2001c, 2001d). It is, in fact, the dynamics equation for three-body motion that Newton could not work out in his lifetime.

We observe that the active force in the three-body motion for \( P \) is a polynomial. This indicates that \( P \) is exerted upon by various action forces, which can be expressed as follows:

\[
F_o = \sum_{i=1}^{3} F_i \quad (i = 1, 2, 3)
\]

(4)

3 Essential Feature of Gravitation

The right side of expression (4) comprises three kinds
of forces: tidal forces, PML force, and gravitation (Liu et al., 2001). The tidal and PML forces have been specifically discussed by authors in other papers. Herein, we will confine ourselves to a detailed discussion of $F_3$, specifically when $i = 3$.

Expression (4) can be further derived as shown in expression (5).

$$F_i = \frac{mL}{R} \cos \theta \left( \frac{d\theta}{dt} \right)^2$$  \hspace{1cm} (5)

where $F_i$ is the active force exertion on passive point $P$ (N), $m$ is the quality of passive point $P$ (kg), $L$ is the distance between the passive point $P$ and fixed point $S$ (m), $I$ is the distance between the passive point $P$ and active point $E$ (m), $R$ is the distance between the passive point $P$ and the geocenter (m), $\theta$ is the angle of the passive point $P$ from the connection line of the other two points ($^\circ$), and $t$ is time (s).

To clearly delineate the expression (5), we make the following transformation.

Generally, $\theta$ is negligible compared with $R$, $L \approx I$; thus, the above expression can be rewritten as shown

$$F_i = \frac{mL^2}{R} \omega^2$$  \hspace{1cm} (6)

Where $\omega$ is the angular velocity when passive point $P$ deviates from the connection line between the central position of the fixed point $S$ and the active point $E$. Substituting $I \omega = v$ into expression (6), this formula can be simplified as in expression (7)

$$F_i = \frac{mv^2}{R}$$  \hspace{1cm} (7)

where $v$ is the motion variation velocity of the connection line between the passive point $P$ and fixed point $S$ as it sweeps the connection line between the active point $E$ and fixed point $S$. Clearly, expression (7) is a typical expression for the centripetal force.

Therefore, expression (5) is the gravitational equation, and $F_i$ is the gravitation of which Newton had not discovered, which drives the passive point $P$ with the active point $E$ to move around the fixed point $S$.

Gravitation is generated by the simultaneous cooperation of a progenitor and a paternal orbital focus. The magnitude of gravitation is not related to the progenitor’s body quality and paternal orbit but is related to the body quality upon which gravitation is exerted (Cao et al., 2013; Bao et al., 2014). Gravitation is not only inversely proportional to the square of the distance, as Newton mentioned. It is only proportional to the square of the distance between the active point and the orbital plane of the paternal orbit, and is inversely proportional to the distance of its own orbit. This explains why some comets can travel very close to some planets but not be attracted by them. This is because the gravitation that brings comets into a specific orbit is jointly caused by the Sun and Milky Way’s core.

For any bodies moving around the Earth, their gravitation is zero only if $\omega = 0$. Clearly, it is only of theoretical significance to say that the body’s gravitation is zero. For any bodies that leave their motion orbit once they return to the solid surface of the Earth, whether in the air, the mountains, or the ground, the Sun (progenitor) loses its gravitational exertion on this body.

Expression (5) indicates that the gravitation of any three-body motion system can be calculated as long as the values of $m$, $\theta$, $R$, $L$, and $I$ are known.

4 Discussions

4.1 On gravitation

Gravitation was discovered by Newton through his observations of the orbital motion of the universe. While it was doubted by Leibniz, it was accepted as a theorem and became known as the universal gravitation law.

Newton delivered his statement about gravitation in the second book he published, Optics, in its second version— For the consideration of that I didn't think gravitation as the essential properties for any object, I am doubt about its reason in a questionable way, because I am not satisfied with it due to lack of experimental data. This similar statement can be seen on page from 350 to 382, was published on July 16, 1717, 31 years after his first definition of gravitation, which indicates that he remained slightly suspicious about the existence of gravitation, after 31 years of observation and contemplation on the subject.

According to Newton’s viewpoint, as expressed in Optics, gravitation was deduced by Newton through comprehensive analysis of the relationship of the satellites and planets, and the planets and stars. The key point of his comprehensive analysis is that an explanation was found and viewed as a principle to explain some phenomenon that occurred as a result of this principle; thus proving the explanation. To prove that gravitation occurs in any body at any time, he refers to occurrences of both the body gravity and the oceanic tides (this is where the tidal generating force comes from) as the foundation of his analysis (Folger, 1970; Mason, 1971; Wyrki and Meyers, 1976; Gill, 1982).

Newton’s original definition on gravitation may be described as follows: it depends on the quantity of particles in a solid substance, and the force will have effect at extremely far distances in all directions, but will gradually decrease and is inversely proportional to the square of the distance. According to Newton’s definition, gravitation occurs between any two objects.
4.2 Occurrence of gravitation

Analysis herein shows that gravitation occurs between bodies and is proportional to their specific qualities and to the size of the angular velocity of their deviation from the center position of the progenitor—paternal orbital plane, and to the square of the distance of the progenitor orbital plane. However, it is inversely proportional to the distance of the paternal orbital plane and is not related to another body’s quality.

However, gravitation, as discussed herein, is based on mathematical principles and the derivation of its expression with a mathematical and logical consideration of the entire process. The gravitation equation coincides with the principle of dimensionless analysis and corresponds with practical conditions. According to this definition, gravitation only occurs among two bodies present in an orbital motion relationship. The occurrence of gravitation simultaneously results from the progenitor and paternal body.

Because this gravitational equation differs from the traditional formula of universal gravitation, it is possible to discuss the specific conditions under which gravitation is equal to the gravity or to zero.

4.3 Gravitational equations

Newton’s concepts were solely based on his observations; moreover, despite a lack of experimental verification, he presented his speculations about gravitation. Whether these speculations are correct could not be ascertained. Therefore, it is understandable that he failed to derive the expression for gravitation in his lifetime.

It would be unreasonable to expect that Newton, who was skilled in mathematical analysis, to leave a proportional coefficient for later researchers to develop. There were dozens of years between his discovery of gravitation and his death. During this period, he and his students faced repeated challenges and suspicion from opponents; furthermore, he would have tried his best to discover the gravitational expression to fight back if he had some idea of how to do so.

The so-called universal gravitation, which was constructed by a later researcher on the basis of Newton’s serial deductions, has no scientific foundation to make the expression conform to dimensional requirements or to manually treat the “gravitation coefficient G” as a physical unit.

Not even once, on the basis of universal gravitational equation, has someone yet found one plane wherein gravitation is zero and along which a spacecraft could fly away.

4.4 Gravitation and gravitational acceleration

When one body deviates from its original orbit into a higher level orbit, the body changes the original three-body motion relationship. The original progenitor body becomes the paternal body and gravitation results from the new progenitor and paternal body.

The process whereby one body loses its gravity is actually a process whereby the influence gradually decreases by the paternal body while gradually increasing by the progenitor body.

When the body deviates from its orbit and returns to the paternal body, the gravitational influence of the progenitor body tends to disappear, and the main result is the gravitation variation from the gravitational acceleration of the paternal body.

Gravitation is always exerted upon a static body on the surface of the Earth in the process of Earth’s rotation, by both the Sun and Earth.

Gravitational acceleration at any point on the Earth is different because of the various times (different orbital position) and different magnitudes $\theta$. Similarly, gravitational acceleration for any bodies at the same time but different locations is different because of their different $R$, $l$, and $l$ values.

The gravitational equation perfectly explains why gravitational acceleration is different at different latitudes on the Earth is different for the same location at different seasons and is different for the North and South Poles. The previous interpretation of the gravitational acceleration for different locations lacks a theoretical basis.

4.5 The honor of discovery of gravitation still belongs to Newton

Newton created the “fluxions method” to analyze scientific problems but did not use it to deduce the gravitational equation.

We have applied Newton’s “fluxions method” to actually deduce the dynamic equation for three-body motion, and have corrected some past mistakes with respect to the gravitation and tidal forces and their relationship.

5 Conclusions and Prospects

5.1 Conclusions

(1) Gravitation simultaneously results from the progenitor and paternal body. For any bodies, their weight loss and gain actually decrease and increase their gravitation, respectively.

(2) The action forces that cause the variation of the motion status for a three-body system are based on the analysis of the Sun, Earth, and Moon’s motion.
(3) The tidal force, the PML Force, and gravitation are a group of action forces that simultaneously occur; moreover, their exertion and reaction relationship, influencing factors, action modes, and motion states are all different.

(4) Force is the reason for changes in the motion state of a body. The action force on a body can be obtained by analyzing the variation of a body’s motion status.

(5) This research has scientific and academic significance, and a complete and persuasive theoretical basis. It also makes precise deductions and follows a logical reasoning process.

5.2 Prospects

The universe is in constant motion in the simplest ways, with single-, two-, and three-body motion. Does four-body motion occur in the universe? This question goes beyond the limitations of the authors’ cognitive capacities.

In this paper, the gravitational equation of nature is presented through the analysis of three-body motion. What kind of exertion situation exists for two-body motion? Our results indicate that the exerted force on the active body by the fixed body is a spherical force, causing the active point to change entirely and periodically in its magnitude and direction.

Exertion analysis for a single body motion shows a series of forces that are temporarily named “constraint forces.”

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Appendix: Derivation of the dynamic formula analysis

The deduction process from expression (1) to (2) is shown as follows.

\[ R^2 = l^2 + \dot{r}^2 - 2L\cos \theta. \]  

(1)

Setting \( R = \sqrt{L^2 + \dot{r}^2 - 2L\cos \theta} \), then

\[ \frac{dR}{d\theta} = \frac{1}{2\sqrt{L^2 + \dot{r}^2 - 2L\cos \theta}} 
  \left[ 2l \frac{dl}{d\theta} + 2L \frac{dL}{d\theta} \right] \left[ \frac{d}{d\theta} \cos \theta + 2l \frac{dL}{d\theta} \sin \theta \right] \]

\[ \frac{dR}{dr} = \frac{1}{\sqrt{L^2 + \dot{r}^2 - 2L\cos \theta}} \left[ l + L \frac{dL}{d\theta} - L \cos \theta - l \cos \theta + L \frac{d\theta}{dt} \sin \theta \right] \]

\[ \frac{dR}{d\theta} = \frac{1}{\sqrt{L^2 + \dot{r}^2 - 2L\cos \theta}} \left[ l + L \frac{dL}{d\theta} - l \cos \theta + L \frac{d\theta}{dt} \sin \theta \right] \]

Setting \( A = l + L \frac{dL}{d\theta} - l \cos \theta + L \frac{d\theta}{dt} \sin \theta \)

(1-1)

Correspondingly, the deduction below is also feasible.

\[ \frac{d\frac{dR}{d\theta}}{d\theta} = \frac{1}{2} \frac{d}{d\theta} \frac{dR}{d\theta} A + \frac{1}{2} \frac{dA}{d\theta} \]

(1-2)

\[ \frac{dA}{d\theta} = \frac{2}{d\theta} \frac{dL}{d\theta} \frac{d\theta}{dt} \]

(1-3)

Therefore, \[ \frac{d\frac{dR}{d\theta}}{d\theta} = \frac{1}{2} \frac{d}{d\theta} \frac{dR}{d\theta} A + \frac{1}{2} \frac{dA}{d\theta} \]

From \( A = l + L \frac{dL}{d\theta} - l \cos \theta + L \frac{d\theta}{dt} \sin \theta \), then

\[ A = (l - L \cos \theta) + (l - L \cos \theta) L + L \frac{d\theta}{dt} \sin \theta \]

\[ \frac{dA}{d\theta} = \frac{d}{d\theta} \left[ (l - L \cos \theta) + (l - L \cos \theta) L + L \frac{d\theta}{dt} \sin \theta \right] \]

\[ \frac{dL}{d\theta} = \frac{d}{d\theta} \left[ (l - L \cos \theta) + (l - L \cos \theta) L + L \frac{d\theta}{dt} \sin \theta \right] \]

The transformation into the time function is shown below.

\[ \frac{dA}{dt} = \frac{d}{dt} \left[ (l - L \cos \theta) + (l - L \cos \theta) L + L \frac{d\theta}{dt} \sin \theta \right] + (l - L \cos \theta) L + L \frac{d\theta}{dt} \sin \theta \]

(1-4)

\[ A^2 = \left[ (l - L \cos \theta) + (l - L \cos \theta) L + L \frac{d\theta}{dt} \sin \theta \right]^2 \]

(1-5)

Therefore, \[ \frac{d\frac{dR}{d\theta}}{d\theta} = \frac{1}{2} \frac{d}{d\theta} \frac{dR}{d\theta} A + \frac{1}{2} \frac{dA}{d\theta} \]

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\[
\frac{d^2 R}{dt^2} = \frac{1}{\sqrt{u}} \left( \frac{dA}{dt} - \frac{1}{u} A^2 \right)
\]  

(1-6)

Substituting \(\frac{dA}{dt}\), \(A^2\), and \(u\) into expressions (1–6), the variation acceleration \(\frac{d^2 R}{dt^2}\) for any spherical particle in the radial direction on Earth would be obtained when it revolves around the Sun. As tested (dimensional test), for the items and their expansion, such as \(\frac{d^2 R}{dt^2}\), \(\frac{1}{\sqrt{u}} \frac{dA}{dt}\), \(\frac{1}{\sqrt{u}u} A^2\), all data items contain acceleration coefficients. Therefore, multiply the quality \(m\) of the particle on both sides, to obtain

\[
\frac{m}{\sqrt{u}} \frac{d^2 R}{dt^2} = m \frac{dA}{dt} - m \frac{1}{u} A^2
\]  

(2)