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Deep Time Reconstructions of Internal Earth Structure with the Adjoint Method

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The adjoint method is a powerful tool to obtain gradient information in a computational model relative to unknown model parameters, allowing one to solve inverse problems where analytical solutions are not available or the cost to determine prohibitive. In geodynamics its most prominent application relates to the restoration problem of mantle convection (i.e., to reconstruct past mantle flow states with dynamic earth models back in time by finding optimal flow histories relative to the current model state, see Fig. 1), so that poorly known mantle flow parameters can be tested explicitly against observations gleaned from the geologic record. By enabling construction of time-dependent, internal Earth structure models, this method has the potential to link present-day and deep-time observations from seismology, mineral physics, geology, and palaeomagnetism simultaneously in a dynamically consistent way, greatly enhancing our understanding of the solid Earth system and its linkage to surface processes as mapped by geologists.

The formal inverse problem of the adjoint approach employs so-called “adjoint equations”. These provide sensitivity information relative to earlier system states. Adjoint equations have been derived for incompressible (Bunge et al., 2003; Ismail-Zadeh et al., 2004; Horbach et al., 2014), compressible (Ghelichkhan and Bunge, 2016) and thermo-chemical mantle flow (Ghelichkhan and Bunge, 2018). Moreover, geodynamicists have related the uniqueness properties of the inverse problem explicitly to the tangential component of a mantle convection model’s surface velocity field (Colli et al., 2015). Knowledge of the latter is essential to assure convergence (Vynnytska and Bunge, 2014) and to obtain a small null

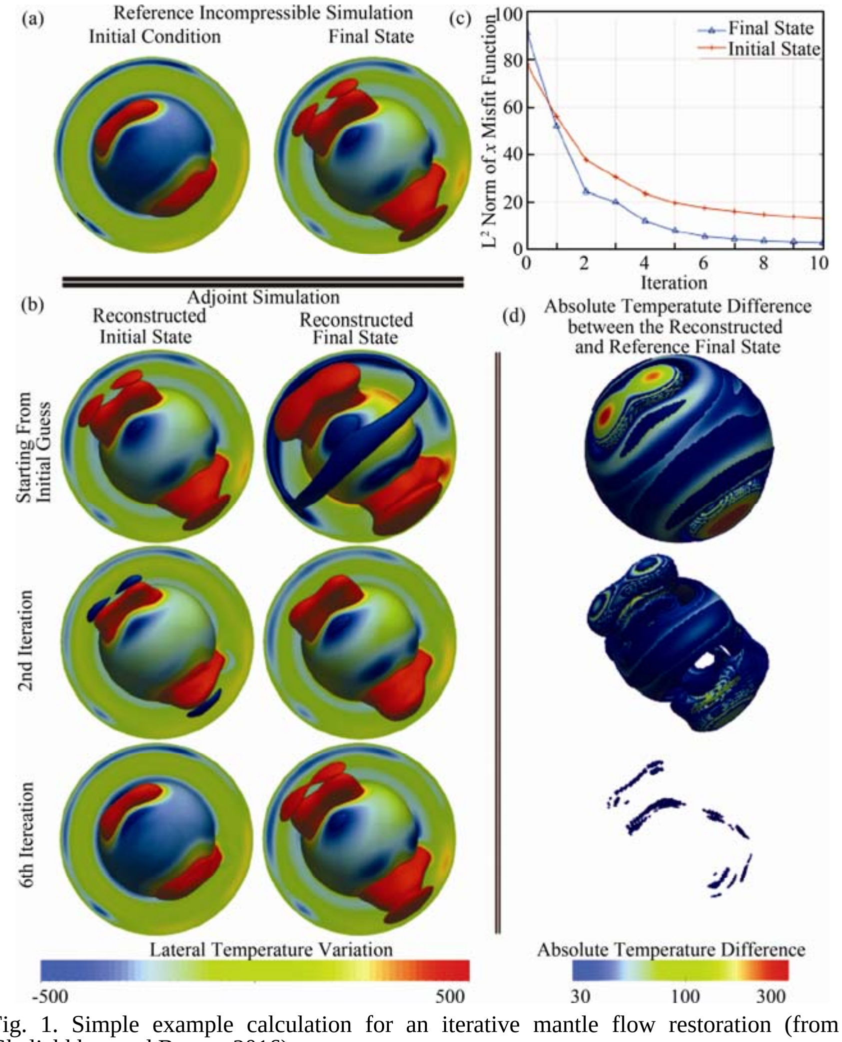

Fig. 1. Simple example calculation for an iterative mantle flow restoration (from Ghelichkhan and Bunge, 2016).
(a) Initial (left) and final (right) state of incompressible reference mantle convection model, with colors (blue cold, red hot) showing temperature. Blue and orange iso-surfaces represent -250 and 250 K, respectively. Two thermal anomalies with excess temperature of 500 K traverse the mantle depth after a transit time. (b) Same as (a) but for restored initial and final state at adjoint iteration zero, two and six, where the final state of the reference convection model serves as first-guess initial condition (top left). Reference model and restored model are visually identical after six adjoint iterations. (c) L 2 norm of misfit function (blue, final state error), and initial state error (red) as function of adjoint iteration. A majority of initial and final state error is reduced in the first two adjoint iterations. (d) Spatial distribution of absolute value of the final state error at adjoint iteration zero (top), two (middle) and six. Error is largest in regions of upwelling plumes. A significant error reduction is achieved after 6 adjoint iterations.

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space for the restored flow evolution. This makes past plate motions the input of retrodictions rather than their output, potentially implying that it is not viable to construct self-consistent models of plate tectonics that are testable against the geologic record.

While horizontal motion of the lithosphere cannot be predicted from mantle flow restoration, it is well known that mantle convection also induces vertical deflections of the Earth's surface (see Braun, 2010; Colli et al., 2016 for recent reviews). This convectively maintained topography was referred early on as “Dynamic Topography” by Hager et al. (1985). Recently, there has been much effort to quantify dynamic topography and its temporal evolution as a link to mantle flow processes (Bunge and Glasmacher, 2018), both in continental regions (e.g., from studies of continental scale stratigraphy, planation surfaces, elevated passive margins, thermochronology), as well as in the oceanic realm where our understanding of plate subsidence as a function of age permits residual depth anomalies to be identified and mapped (Hoggard et al., 2017). In this presentation we review current state-of-the-art of mantle flow retrodictions (Colli et al., 2018), and their link to vertical lithosphere motions. We present the first global mantle flow retrodictions for geodynamically plausible, compressible, extremely high-resolution Earth models with more than 670 million finite elements, going back in time to the Mid-Paleogene. These models are computationally intensive, and need weeks of integration time on thousands of processors resident in the German High Performance Computing Center (LRZ) in Munich. Our mantle flow retrodictions involve the dynamic effects from a low viscosity zone (LVZ) in the upper mantle, assimilate a past plate motion model for the tangential surface velocity field, and probe the influence from uncertain geodynamic modeling parameters using two different state estimates for the present-day mantle heterogeneity structure as imaged by two recent global seismic tomographic studies, and two different values for deep-mantle viscosity. Focusing on the African hemisphere, we found that our retrodictions produced a spatially and temporally highly variable asthenosphere flow with faster-than-plate velocities, and a history of dynamic topography variations characterized by local doming events. These results agree with published considerations of plate driving forces, and regional scale uplifts. Our results suggest that improved constraints on non-isostatic vertical motion of Earth’s surface—provided, for instance, by basin analysis, seismic stratigraphy, landform studies, or the sedimentation record—will play a key role in improving understanding of Recent mantle flow history and its link to geological surface processes.

Key words: adjoint method, inverse problems, paleo mantle convection, flow restorations, dynamic topography

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