3D MT anisotropic inversion based on finite-element method with unstructured grids

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Introduction

Magnetotelluric (MT) method has been widely used in the investigation of the deep-earth structures. Electrical anisotropy is frequently observed in practice from MT data, especially when studying the deep-earth structures. Conventional 3D MT forward modeling are mostly based on structured grids, while the inversions generally assume an isotropic model. However, the structured method cannot accurately model the complex geology like topography, and incorrect results are obtained when using isotropic models to do the interpretation. Finite-element (FE) method are capable of using unstructured grids and have been successfully used in 3D MT modeling for anisotropic media with topography (Cao et al, 2018). In the paper, FE method and the limited memory quasi-Newton method (L-BFGS) for optimization are used to effectively invert the complex structures of the earth including anisotropy.

Forward and inversion method

For 3D MT problem, the vector Helmholtz equation for the electric field can be written as

$$\nabla\times\nabla\times\mathbf{E}+i\omega\mu_0\underline{\underline{\sigma}}\mathbf{E}=0\,, \tag{1}$$

where $\underline{\underline{\sigma}}$ denotes the anisotropic conductivity tensor.

The fully anisotropic modeling based on eq. (1) with the unstructured finite-method is accomplished by Cao et al. (2018). And the weak form of eq. (1) can be written as

$$\iiint_\Omega(\nabla\times\mathbf{E})\cdot(\nabla\times\mathbf{\Phi})\mathrm{d}v+\iiint_\Omega i\omega\mu_0\underline{\underline{\sigma}}\mathbf{E}\cdot\mathbf{\Phi}\mathrm{d}v=\iint_{\partial\Omega}\mathbf{\Phi}\cdot(\mathbf{n}\times(\nabla\times\mathbf{E}))\mathrm{d}s\,, \tag{2}$$

where $\mathbf{n}$ is the normal vector on outer boundary $\partial\Omega$. We apply the Dirichlet boundary conditions on the top boundary of Ω. Eq. (2) can be transformed into a complex system of linear equations by which we can solve the 3D MT anisotropic forward problem. In this paper, we focus our attention on a 3-axial anisotropic model.

We define the following objective functional for 3D MT anisotropic inversion:

$$\varphi(\boldsymbol{m})=\varphi_d(\boldsymbol{m})+\lambda\varphi_m(\boldsymbol{m})\,, \tag{3}$$

where $\boldsymbol{m}$ is the model conductivity vector containing three principal axis conductivity, λ is the regularization parameter, $\varphi_d(\boldsymbol{m})$ is the data misfit. The model constraint $\varphi_m(\boldsymbol{m})$ can be written as

$$\begin{aligned}\varphi_m(\boldsymbol{m})=&(\boldsymbol{m}_x-\boldsymbol{m}_{x0})^T\mathbf{W}^T\mathbf{W}(\boldsymbol{m}_x-\boldsymbol{m}_{x0})+(\boldsymbol{m}_y-\boldsymbol{m}_{y0})^T\mathbf{W}^T\mathbf{W}(\boldsymbol{m}_y-\boldsymbol{m}_{y0})\\&+(\boldsymbol{m}_z-\boldsymbol{m}_{z0})^T\mathbf{W}^T\mathbf{W}(\boldsymbol{m}_z-\boldsymbol{m}_{z0})\end{aligned}\,, \tag{4}$$

where $\boldsymbol{m}_x$, $\boldsymbol{m}_y$ and $\boldsymbol{m}_z$ are the principal axis conductivity of $\boldsymbol{m}$, while $\boldsymbol{m}_{x0}$, $\boldsymbol{m}_{y0}$ and $\boldsymbol{m}_{z0}$ are the corresponding prior model, $\mathbf{W}$ is a first order of spatial finite-difference matrix. The L-BFGS method (Nocedal ,1980) is used to optimize eq. (3) which avoids explicit calculation of Hessian or Jacobean matrices, saving the memory and computational time.

Synthetic data inversions

To test the adaptation of our algorithm to anisotropic MT data influenced by topography, the model in Fig.1 is set up. Seven frequencies logarithmically distributed between 0.01Hz and 10 Hz are used for the synthetic data and 2% Gaussian noises are added. We use half-space of 100Ω·m as the initial model. The anisotropic inversion results are plotted in Fig.2. One can see that the distributions of the inverted ρ_x and ρ_y match the location of true model well on the condition of a trapezoidal hill topography. But ρ_z

is not resolvable as described in Yin (2003) due to the nature of plane wave.

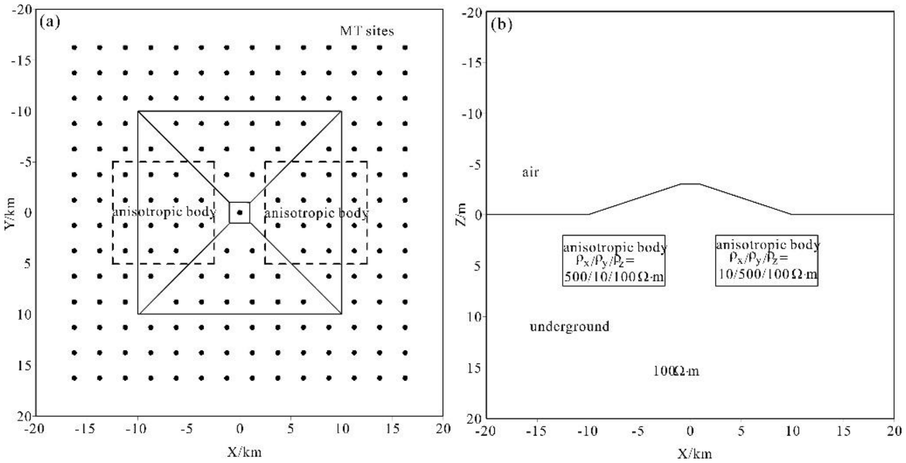

Figure.1 Schematic display of the synthetic model. (a) Plane view; (b) Section view

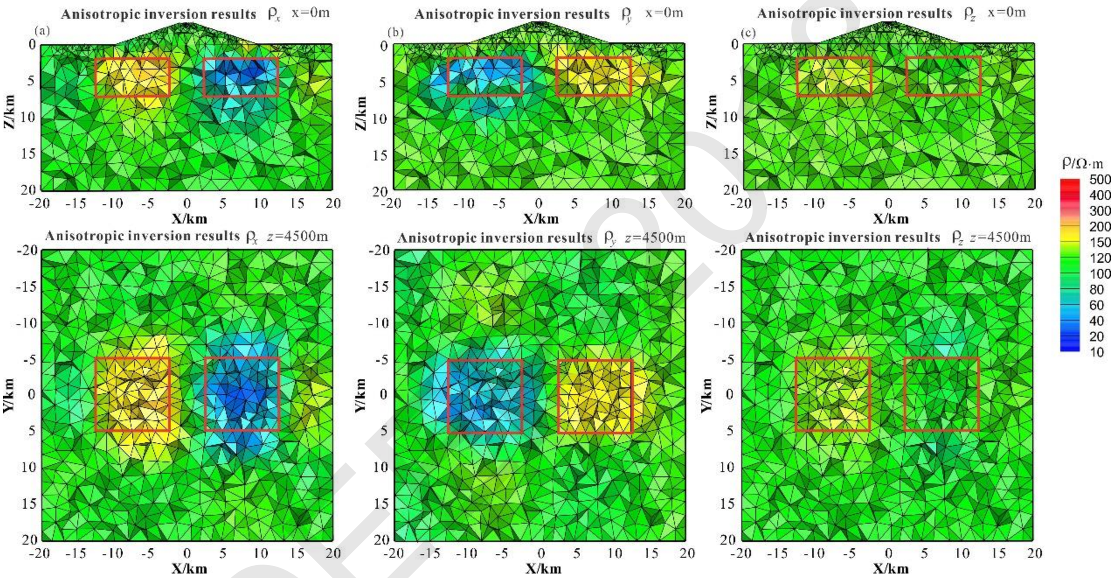

Figure.2 The anisotropic inversion results of the synthetic model. (a) and (b) Principle resistivity ρ_x at xz- and xy-section; (c) and (d) Principle resistivity ρ_y at xz- and xy-section; (e) and (f) Principle resistivity ρ_z at xz- and xy-section.

Conclusions

Numerical experiments show that unstructured finite-element method with L-BFGS method can efficiently invert 3D MT data with topography directly without additional correction. MT inversions for an anisotropic earth can resolve the resistivities in the horizontal principal axis directions, however the resistivity in the vertical direction is non-resolvable.

Reference

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