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Pulsation Solution to the Equation of Earth's Gravitational Field (Main Outcome)

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Abstract Using d'Alembert equation as the approximation of Einstein's equation, a solution is given in this paper to the time-dependent gravitational equation of the Earth in consideration of the Earth's features, which describes the characteristics of pulsation of the Earth and the structures of spherical layers of its interior, thus providing a theoretical basis for establishing the idea of mantle pulsation.

Key words: d'Alembert equation, formula of gravitational field, spherical layers, density model, mantle pulsation

When the author proposed his theory of the pulsation of the Earth in 1981 (Jiang Zhi, 1982), an academic problem concerning the gravitational field of the Earth was involved, but this problem has not been solved theoretically so far. The present paper attempts to solve this particular problem.

When dealing with space-time variation of the gravitational field of the Earth, it is necessary to employ the time-bearing equation of the gravitational field, namely Einstein's equation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{C^4} T_{\mu\nu} \qquad (1)$$

The approximate expression (Fock, 1965) of Formula (1) runs as follows:

$$\Delta\overline{U} - \frac{1}{C^2}\frac{\partial^2 \overline{U}}{\partial t^2} = -4\pi G(C^2 + \frac{1}{2}U)T^{00} \qquad (2)$$

where, $\overline{U} \approx U$, $T^{00} \approx \sigma / C^2$ and σ is the mass density.

The d'Alembert equation (3) may be drawn out of Formula (2) in consideration of the following characteristics of the Earth:

(1) the Earth's gravitational potential is by far weaker than the square of the light velocity;

(2) most probably, the gravitational potential and mass density of the Earth share the same change with time;

(3) the gravitational potential and mass density of the Earth probably have the same variation with longitude and latitude;

(4) at the Earth's core, the gravitational potential is zero, but the mass density is greater than zero; outside the solid Earth, the potential is greater than zero, but the density is close to zero;

(5) at positions remote from the Earth, the gravitational potential should approach to Newton's potential.

$$\Delta U - \frac{1}{\lambda^2}\frac{\partial^2 U}{\partial \zeta^2} = 4\pi G\sigma \qquad (3)$$

where $\lambda = C / v; \zeta = \int v\, dt$ and the corresponding solutions to Formula (3) may be written as follows:

at positions outside the solid Earth,

$$U = \frac{GM}{r}\left[1 + \sum_{l=2}^{\infty}\sum_{m=0}^{l}\left(\frac{R_0}{r}\right)^l Y_{lm}(\theta,\varphi)\right]\left[1 + \sum_{j=1}^{\infty} A_j \cos j\left(\zeta - \frac{r}{\lambda}\right)\right] \qquad (4)$$

at positions interior the solid Earth,

$$U = \frac{GM}{R_0}\sum_{n=1}^{n_r}\sum_{l=0}^{n-l}\sum_{m=0}^{l} R_{nl}(r) Y_{lm}(\theta,\varphi)\left[1 + \sum_{j=1}^{\infty} A_j \cos j(\zeta - \frac{R_0}{\lambda})\right] \qquad (5)$$

where, $Y_{lm}(\theta, \Phi)$ is the spherical function; $R_{nl}(r)$ is as follows:

$$R_{nl}(r) = \left(\frac{2R_0}{na_0}\right)^l \left(\frac{r}{R_0}\right)^{l+1} e^{-\frac{r-R_0}{na_0}} \sum_{i=0}^{n-l-1} \left(\frac{2r}{na_0}\right)^i \frac{(n-l-1)!(2l+1)!}{i!(n-l-1-i)!(2l+1+i)!} \quad (6)$$

whose condition for reaching the maximum value when $l=n-1$ is

$$r = n^2 a_0 \quad (7)$$

Distances of the boundaries of spherical layers from the Earth's core are expressed as:

$$r = (n^2 - 1)a_0 \quad (8)$$

When $a_0=R_0/15$, formula (8) proves to be none other than the empirical discovery (Jiang Zhi, 1982) made known by the author in 1981, that is, for the Earth's core, $r=0$; for the boundary between the interior and exterior core, $r=3a_0$; for the boundary between Earth's core and mantle, $r=8a_0$; and for the Earth's surface, $r=15a_0$. What is very interesting is that by making use of this particular result one can show the distance and mass density σ of Earth's core as follows:

$$r = (k^2 - 1)R_0/15, \qquad 1 \le k \le 4 \quad (9)$$

$$\sigma = \bar{\sigma}\left[a_n - b_n\left(\frac{k^2 - n^2}{2n+1}\right)^2\right], \quad n \le k \le n+1, \quad n=1, 2, 3 \quad (10)$$

where $\bar{\sigma}$ =5.520g/cm^3, representing the average density of the Earth; a_1=2.346, b_1=0.0555; a_2=2.188, b_2=0.402; a_3=1.000, b_3=0.424. Fig. 1 shows that Formula (10) conforms to the present Earth mass-density model (Zhang Jiacheng and Li Wenfan, 1986). Moreover, the Earth mass M and rotational inertia I can be calculated using Formulae (9) and (10):

$$M = \iiint \sigma dv = MJ_0 \quad (11)$$

$$I = \iiint \sigma r^2 \sin^2 \varphi dv = MR_0^2 J_1 \quad (12)$$

where

$$J_0 = \frac{6}{15^3}\sum_{n=1}^{3}\int_n^{n+1} k(k^2-1)^2 \left[a_n - b_n\left(\frac{k^2-n^2}{2n+1}\right)^2\right]dk \quad (13)$$

$$J_1 = \frac{4}{15^5}\sum_{n=1}^{3}\int_n^{n+1} k(k^2-1)^4 \left[a_n - b_n\left(\frac{k^2-n^2}{2n+1}\right)^2\right]dk \quad (14)$$

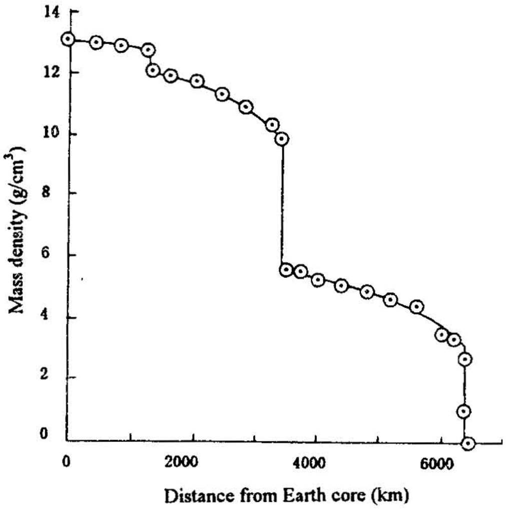

Fig. 1. Earth mass-density model.

Calculations show that J_0=1−0.0022 and J_1= 0.3308 (1+0.0021), of which 0.3308 was obtained at the Earth satellite. Thus, it can be seen that the Earth mass and rotational inertia calculated out of Formula (10) have a deviation of only about two thousandths. What is more, the variation of rotational inertia of the Earth, according to Formula (12), may be represented by the variation of its average radius, and then Formula (14) becomes a certain kind of simplified condition, under which one can discuss the variation of the shape and pulsation of the Earth (Jiang Zhi, 1982, 1996a, 1998).

Under the condition of Earth surface, the previous work of the author concentrated mainly on the discussion of wave number, ξ, of the fundamental wave :

$$\xi = \frac{1}{2\pi}(\zeta - \frac{R_0}{\lambda}) \quad (15)$$

Two patterns concerning ξ were given by the author (Jiang Zhi, 1982, 1992, 1996a, 1996b, 1998), that is, the pattern for the influence of density wave in the Galactic system and the pattern for radioactive energy laid up within the Earth, based on which the effect due to pulsation of the Earth was dealt with and the theoretical geologic time scales (Jiang Zhi, 1982, 1996a, 1998) were first established in the world in 1981.

Taking into consideration of tidal friction and using Formulae (10) and (12), a formula as regards the change of the rotation of the Earth was set up (Jiang Zhi, 1982, 1996a); a model as regards the change of the Earth shape was established (Jiang Zhi, 1996a) based on the above formula, data and information concerning small celestial bodies and some considerations related to the time when the Earth took shape; a formula concerning the palaeotemperature variation was proposed in 1983 (Jiang Zhi, 1983, 1996a) based on the formula in relation to the change of the rotation of the Earth, precession formula of celestial mechanics and Stefan-Boltzmann's law related to thermodynamics; in accordance with the variation of the horizontal field of force in the lithosphere arising from the change in the rotation and shape of the Earth, the author discussed plate tectonics, structural system, wavy mosaic structure, polycyclic development of geosynclines, fracture structural system, cycles of magma-structure and marine regression and transgression, problem of regional stress, problem of continental crust growth, problem of Diwa (Geodepression, or continental depression) and put forward the eight-form pattern for the development of the Earth crust (Jiang Zhi, 1982, 1984, 1992, 1996a, 1996b).

We all know that radioactive elements largely spread in both mantle and crust. If accumulation of radioactive energy within the Earth is one of the important causes for the Earth pulsation, then differential rotation will inevitably occur between the inner core and the mantle due to the fact that mantle pulsation functions as the predominant Earth pulsation. Because between the two layers of the Earth lies the liquid metallic exterior core, which not only forms a condition for core-mantle differential rotation, but also, together with the solid metallic interior core, brings about a condition for the generation of magnetic fields and reversal of magnetic fields based. With the aid of this concept, the author set up for the first time the theoretical geologic time scale of palaeomagnetic polarity in the world in 1983 (Jiang Zhi, 1984, 1996a, 1998). It should be pointed out that the hypothesis of core-mantle differential movement was confirmed thirteen years later by Mr. Song Xiaodong and others in their earthquake observation (Song and Richards, 1996).

Evidence could also be found to verify the existence of mantle pulsation in Earth movement of the recent times. Enlightened by the work conducted by Ma Zongjin, we used the following method to process the data of the major earthquakes above 8 on Richter scale taking place in the Earth since 1800 (Zhang Jiacheng and Li Wenfan, 1986): T represents the sliding accumulative number of earthquakes happening in the world for the past 11 years; O is the sliding accumulative number of earthquakes happening in the circum-Pacific belt for the past 11 years; $C=1-O/T$ is equivalent to the sliding accumulative ratio of earthquakes occurring in the continent for the past 11 years, thus the relationship between C and time t is illustrated in Fig. 2, which shows the time representing the maximum value (years represented by solid line) and the minimum value (years represented by dotted line) of mantle pulsation of k=24.

Fig. 2 suggests that the maximum value of mantle pulsation corresponds to the saddle-peak of the earthquake ratio in the continent, while the minimum value corresponds to the valley bottom of the earthquake ratio in the continent or the peak value of the earthquake ratio in the circum-Pacific belt.

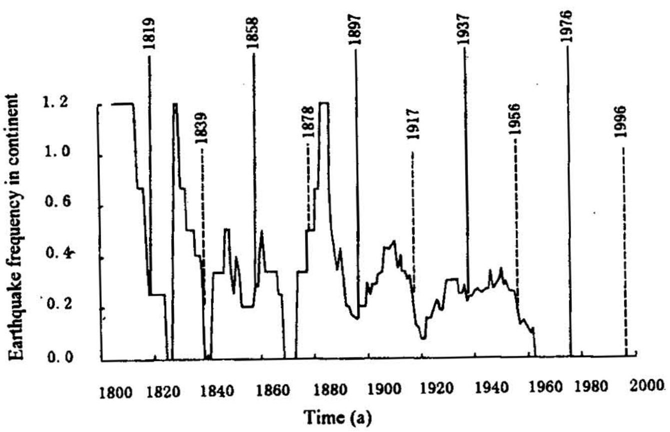

Fig. 2. Earthquake frequency in the continent.

Because continents moved towards the poles when the Earth mantle expanded and drifted towards the equator when the mantle contracted; but stood relatively still when Earth mantle pulsation reached its climax and in the meantime more earthquakes took place in moving continents while less earthquakes in still continents, so the saddle-peak of the continent earthquake ratio corresponds to the maximum of mantle pulsation. On the other hand, when the mantle contracted in the vicinity of its minimum point, earthquake frequency went up due to the reduction of Earth's radius and the ever growing oceanic crust would speed up its plunging into the geosyncline, just like the circum-Pacific belt, so the peak of the earthquake ratio corresponds to the minimum of mantle pulsation.

As the cycle of the recent Earth mantle pulsation of k=24 turns out to be 39.2 years (Jiang Zhi, 1982, 1983, 1984, 1996a) and each saddle-peak shows two peaks and one valley, then the overall peak-valley cycle comes approximately to 19.6 years. It can be also concluded from Fig. 2 that 1966, 1976 and 1986 are typical years corresponding to saddle-peaks of the earthquake ratio in the continent; 2005, 2015 and 2025 are typical years corresponding to saddle-peaks of another earthquake ratio in the continent, and so on.

In one word, Formula (5) proves to be the theoretical basis and mathematical prerequisite for the Earth pulsation or Earth mantle pulsation.

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Jiang Zhi Born in February, 1939; graduated from the Department of Geochemistry, University of Science and Technology of China in 1964. He has devoted himself to the research on the hypothesis of pulsation of the earth, statistical geochemistry, the process of cognition and process of economy for 36 years and has written more than 60 papers and published 8 monographs.